REFERENCE PAGES

ALGEBRA

ARITHMETIC OPERATIONS

$a(b+c) = ab + ac$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

EXPONENTS AND RADICALS

$x^m x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$x^{-n} = \frac{1}{x^n}$

$(xy)^n = x^n y^n$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$

$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

FACTORING SPECIAL POLYNOMIALS

$x^2 - y^2 = (x+y)(x-y)$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

BINOMIAL THEOREM

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{1\cdot 2\cdot 3\cdot \cdots \cdot k}$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$

$|x| < a$ means $-a < x < a$

$|x| > a$ means $x > a$ or $x < -a$

GEOMETRY

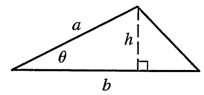
GEOMETRIC FORMULAS

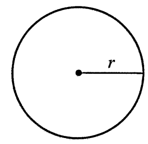
Formulas for area A, circumference C, and volume V:

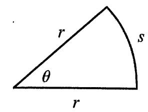
Triangle
$A = \frac{1}{2}bh$
$= \frac{1}{2}ab\sin\theta$

Circle
$A = \pi r^2$
$C = 2\pi r$

Sector of Circle
$A = \frac{1}{2}r^2\theta$
$s = r\theta$ (θ in radians)



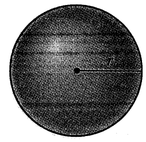


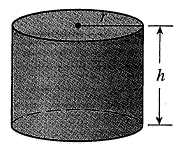


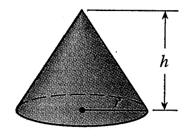
Sphere
$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$

Cylinder
$V = \pi r^2 h$

Cone
$V = \frac{1}{3}\pi r^2 h$
$A = \pi r\sqrt{r^2 + h^2}$







DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

CIRCLES

Equation of the circle with center (h, k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

TRIGONOMETRY

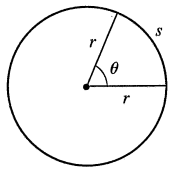
ANGLE MEASUREMENT

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad$ 1 rad $= \dfrac{180°}{\pi}$

$s = r\theta$

(θ in radians)

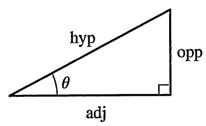


RIGHT ANGLE TRIGONOMETRY

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

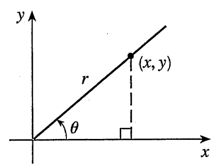


TRIGONOMETRIC FUNCTIONS

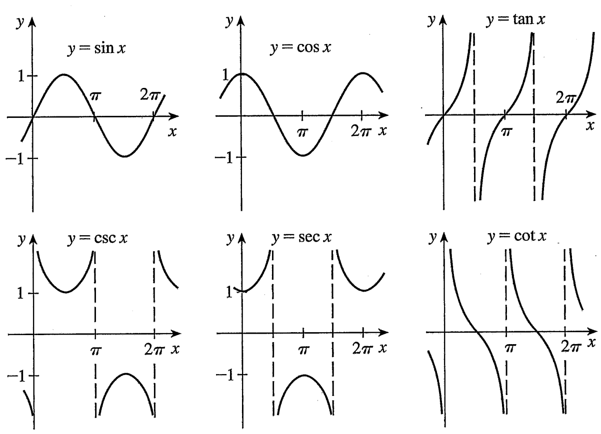
$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



GRAPHS OF TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS OF IMPORTANT ANGLES

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

FUNDAMENTAL IDENTITIES

$\csc\theta = \dfrac{1}{\sin\theta} \qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

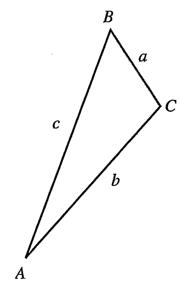
$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

THE LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



THE LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

ADDITION AND SUBTRACTION FORMULAS

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

DOUBLE-ANGLE FORMULAS

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

HALF-ANGLE FORMULAS

$\sin^2 x = \dfrac{1 - \cos 2x}{2} \qquad \cos^2 x = \dfrac{1 + \cos 2x}{2}$

Steps for Finding the Sign of a Function $f(x)$

- Simplify the function $f(x)$.
- Find the domain of $f(x)$. (Find the x values at which f is not defined and omit them from the real number line.)
- Find zeros of $f(x)$, or the solutions of $f(x) = 0$.
- Draw a real number line and mark off numbers at which $f(x)$ is zero or undefined (x values omitted from the domain).
- The last step divides the real number line to several intervals.
- Select a number in each interval and evaluate $f(x)$ at the number
  - If the value of f is positive, then $f(x)$ is positive for all numbers in that interval.
  - If the value of f is negative, then $f(x)$ is negative for all numbers in that interval.

Steps for Drawing the Graph of a Rational Function $f(x)$

- Factor both numerator and denominator.
- Find the domain of $f(x)$. (All x values for which the denominator is not zero.)
- Write $f(x)$ in its lowest terms: $f(x) = \frac{P(x)}{D(x)} = \frac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0}$, Deg $P(x) = n$ and Deg $D(x) = m$.
- If a factor $x - c$ is cancelled in the last step, then $(c, \frac{P(c)}{D(c)})$ is a hole in the graph.
- Find its x- and y-intercepts by setting $P(x) = 0$ and solving for x and finding the $f(0)$ value, respectively.
- Find its vertical asymptotes, if any.
  - Solve $D(x) = 0$. The solutions $x = h$ are the vertical asymptotes.
- Find its horizontal or oblique asymptote.
  - If $n < m$, then the horizontal asymptote is $y = 0$.
  - If $n = m$, then the horizontal asymptote is $y = \frac{a_n}{b_m}$ where a_n and b_m are the leading coefficients of $P(x)$ and $D(x)$, respectively.
  - If $n > m$, then there is an oblique asymptote $y = Q(x)$ where $Q(x)$ is the quotient of the long-division: $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$.
- Determine if the graph crosses the horizontal or oblique asymptote by solving $f(x) = 0$ (x-intercepts), $f(x) = \frac{a_n}{b_m}$, and $R(x) = 0$ for each of the above three cases, respectively.
- Graph the asymptotes and x- and y-intercepts.
- The x-intercepts and vertical asymptotes divide the x-axis into several intervals.
- Choose one or more x values in each interval and evaluate $f(x)$ at those numbers and plot all such points.
- It might be necessary to plot additional points to determine the behavior of the graph near each asymptote.
- In each interval, draw smooth curves connecting plotted points.